Topological Textures in a Ferromagnet-Superconductor Bilayer

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(Received 23 July 2001; published 14 December 2001)

The homogeneous state of a ferromagnet-superconductor bilayer (FSB) with the magnetization perpendicular to the layer can be unstable with respect to the formation of vortices in the superconducting layer. The developing topological instability in the FSB leads to formation of domains in which the direction of the magnetization in the magnetic film and the direction of vorticity in the superconducting film alternate together. This is a new, combined topological structure, which does not appear in isolated layers. Equilibrium domains can appear in the FSB even if the single magnetic film is in a monodomain state.

DOI: 10.1103/PhysRevLett.88.017001 PACS numbers: 74.60.Ge, 74.25.Dw, 74.25.Ha

Heterogeneous ferromagnetic-superconducting structures such as arrays of ferromagnetic dots on top of a superconducting film or ferromagnet-superconductor bilayers, have recently become a focus of studies both experimental [1,2] and theoretical [3–7]. The interest is motivated not only by the important technological promise of devices whose transport properties can be easily tuned by comparatively weak magnetic fields, but also by the appeal of dealing with a new class of physical systems where the interaction between the two order parameters does not suppress either of them. The remarkable properties of the aforementioned systems are due to the magnetic field generated by magnetic dots inside a superconductor and to resulting peculiar vortex structures. Lyuksyutov and Pokrovsky noticed [4] that in a bilayer consisting of homogeneous superconducting (SC) and ferromagnetic (FM) (with the magnetization normal to plane) films, separated by a thin oxide insulator layer in order to avoid proximity effect, SC vortices occur spontaneously in the ground state, despite magnetization does not generate magnetic field in the SC film. In the present Letter, we develop a theory of such vortex-generation instability and the resulting *vortex textures.* We find that the resulting equilibrium distribution of vortices and magnetization is inhomogeneous, and that vortices and magnetization together form a system of alternating domains. We show that the vortex density is higher near the domain walls and that the attraction of vortices to domain walls is so strong that the texture-instability threshold is well below the threshold for an isolated vortex formation. The described texture represents a new class of topological defects which does not appear in isolated SC and FM layers.

We start with refining arguments establishing topological instability in the ferromagnet-superconductor bilayer (FSB) [4]. Assume the magnetic anisotropy to be sufficiently strong to keep magnetization exactly perpendicular to the film (in the z direction). The homogeneous FM film creates no magnetic field outside itself, similar to the electric capacitor, and hence does not alter the state of the SC film. On the other hand, a single vortex in the superconducting film (the so-called Pearl vortex [8]) carries magnetic flux $\Phi_0 = \pi\hbar c/2e$, where c is the light velocity and e is the electron charge. The magnetic field generated by the vortex interacts with the magnetization $\mathbf{m}$ of the FM film decreasing the total energy over $-m\Phi_0$ at a proper sign of vorticity where m is magnetization per unit area of the FM film. The energy necessary to create the Pearl vortex in the isolated SC film is $\epsilon_v^{(0)} = \epsilon_0 \ln(\lambda/\xi)$ [8], where $\epsilon_0 = \Phi_0^2/16\pi^2\lambda$, $\lambda = \lambda_L^2/d$ is the effective penetration depth [9], λ_L is the London penetration depth, and ξ is the coherence length. Thus, the total energy of a single vortex in the FSB is

$$\epsilon_v = \epsilon_v^{(0)} - m\Phi_0 , \tag{1}$$

and the FSB becomes unstable with respect to spontaneous formation vortices as soon as ϵ_v turns negative. Note that, close enough to the SC transition temperature T_c, ϵ_v is definitely negative since the SC electron density n_s and, therefore, $\epsilon_v^{(0)}$ is zero at T_c. At m value small such that $\epsilon_v > 0$ at $T = 0$, the instability exists in the temperature interval $T_v < T < T_c$, where $\epsilon_v(T_v) = 0$, otherwise instability persists until $T = 0$.

A newly appearing vortex phase cannot consist of the vortices of one sign. Indeed, any system with the average vortex density n_v would generate a constant magnetic field $B_z = n_v\Phi_0$ along the z direction. The energy of this field for a finite film of the linear size L_f would have grown as L_f^3, quickly exceeding the gain in energy due to creation of vortices proportional to L_f^2. Hence, in order for the vortex array to survive, the film should split in domains with alternating magnetization and vortex directions. We show below that, if the domain linear size L is much greater than the effective penetration length λ, the most favorable arrangement is the stripe domain structure (see Fig. 1).

To this end, we write the total energy of the bilayer in the form

$$U = U_{sv} + U_{vv} + U_{vm} + U_{mm} + U_{dw} , \tag{2}$$

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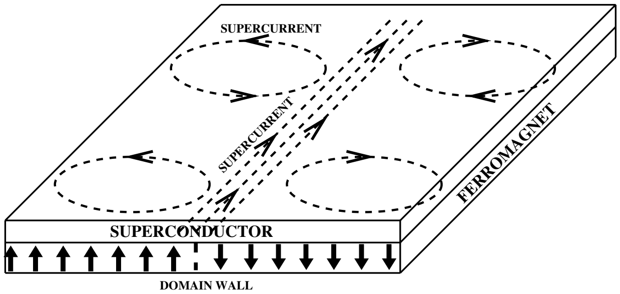

FIG. 1. Magnetic domain wall and coupled arrays of superconducting vortices with opposite vorticity. Arrows show the direction of the supercurrent.

where U_{sv} is the energy of single vortices; U_{vv} is the vortex-vortex interaction energy; U_{vm} is the energy of interaction between the vortices and the magnetic field generated by domain walls; U_{mm} is the self-interaction energy of the magnetic layer; U_{dw} is the linear tension energy of the domain walls. We assume the 2D periodic domain structure to consist of two equivalent sublattices, so that the magnetization $m_z(\mathbf{r})$ and density of vortices $n(\mathbf{r})$ alternate when crossing from one sublattice to another. Magnetization is supposed to have a constant absolute value: $m_z(\mathbf{r}) = ms(\mathbf{r})$, where $s(\mathbf{r})$ is the periodic step function equal to $+1$ at one sublattice and -1 at the other one. We consider a dilute vortex system where vortex spacing is much larger than λ. Then the single-vortex energy with

$$U_{sv} = \epsilon_v \int n(\mathbf{r})s(\mathbf{r})\, d^2x \,. \tag{3}$$

Because of "average neutrality" of the periodic stripe system, the energy of a single vortex in Eq. (3) is different from (1): $\epsilon_v = \epsilon_v^{(0)} - m\Phi_0/2$. The vortex-vortex interaction energy is

$$U_{vv} = \frac{1}{2} \int n(\mathbf{r})V(\mathbf{r} - \mathbf{r}')n(\mathbf{r}')\, d^2x\, d^2x', \tag{4}$$

where $V(\mathbf{r} - \mathbf{r}')$ is the pair interaction energy between vortices located at points $\mathbf{r}$ and $\mathbf{r}'$. Its asymptotics at large distances $|\mathbf{r} - \mathbf{r}'| \gg \lambda$ is $V(\mathbf{r} - \mathbf{r}') = \Phi_0^2/(4\pi^2|\mathbf{r} - \mathbf{r}'|)$ [10]. This long-range interaction is induced by the magnetic field generated by the Pearl vortices and their slowly decaying currents. The energy of vortex interaction with the magnetic field generated by the magnetic film looks as follows [5]:

$$U_{vm} = -\frac{\Phi_0}{8\pi^2\lambda} \int \nabla\varphi(\mathbf{r} - \mathbf{r}')n(\mathbf{r}') \cdot \mathbf{a}^{(m)}(\mathbf{r})\, d^2x\, d^2x'. \tag{5}$$

Here $\varphi(\mathbf{r} - \mathbf{r}') = \arctan\frac{y-y'}{x-x'}$ is a phase shift created at a point $\mathbf{r}$ by a vortex centered at a point $\mathbf{r}'$, and $\mathbf{a}^{(m)}(\mathbf{r})$ is the value of the vector potential induced by the FM film upon the SC one. The magnetic self-interaction reads

$$U_{mm} = -\frac{m}{2} \int B_z^{(m)}(\mathbf{r})s(\mathbf{r})\, d^2x \,. \tag{6}$$

Finally, the domain wall linear energy is $U_{dw} = \epsilon_{dw}L_{dw}$, where ϵ_{dw} is the linear tension of the domain wall and L_{dw} is the total length of the domain walls.

Let us analyze the vortex-domain-wall interaction U_{vm}. Magnetic vector-potential $\mathbf{A}^{(m)}$ obeys the London-Pearl magnetostatic equation:

$$\nabla \times (\nabla \times \mathbf{A}^{(m)}) = \left[-\frac{1}{\lambda}\, \mathbf{a}^{(m)} + 4\pi\nabla \times [\hat{z}m(\mathbf{r})] \right] \delta(z) \,. \tag{7}$$

We consider $L \gg \lambda$; therefore the term $\nabla \times (\nabla \times \mathbf{A}^{(m)})$ is negligible and we find

$$\mathbf{a}^{(m)} \approx -4\pi m\lambda \hat{z} \times \nabla s(\mathbf{r}) \,. \tag{8}$$

The phase gradient entering Eq. (5) can be transformed as $\nabla\varphi(\mathbf{r}) = \hat{z} \times \nabla \ln|\mathbf{r} - \mathbf{r}'|$. Plugging this expression into (5), integrating by part, and employing relation $\nabla^2 \ln|\mathbf{r} - \mathbf{r}'| = -2\pi\delta(\mathbf{r} - \mathbf{r}')$, we arrive at a following result:

$$U_{vm} = -\phi_0 \int m(\mathbf{r})n(\mathbf{r})\, d^2x \,. \tag{9}$$

This result implies that the vortex-domain-wall interaction renormalizes the single-vortex energy turning it into $\tilde{\epsilon}_v = \epsilon_0 - 3m\Phi_0/2$. Thus, the term U_{vm} can be removed from the total energy (2) and the single-vortex contribution U_v must be substituted by $\tilde{U}_v$, which differs from (3) by replacement of ϵ_v by $\tilde{\epsilon}_v$. In physical terms, it means that the vortex attraction to the domain walls lowers the threshold of the spontaneous appearance for the vortex-domain structure.

The next step is the minimization of energy over the vortex density. It appears only in the two first terms of the total energy. Their sum can be conveniently denoted $U_v \equiv \tilde{U}_{sv} + U_{vv}$. To simplify the minimization, we apply the Fourier expansion of the periodic functions: $s(\mathbf{r}) = \sum_{\mathbf{G}} s_{\mathbf{G}} e^{i\mathbf{G}\mathbf{r}}$ and $n(\mathbf{r}) = \sum_{\mathbf{G}} n_{\mathbf{G}} e^{i\mathbf{G}\mathbf{r}}$. The energy U_v in Fourier representation reads

$$U_v = \sum_{\mathbf{G}} \left(\tilde{\epsilon}_v s_{\mathbf{G}} n_{-\mathbf{G}} + \frac{1}{2} V_{\mathbf{G}} n_{\mathbf{G}} n_{-\mathbf{G}} \right), \tag{10}$$

where $V_{\mathbf{G}} = \int V(\mathbf{r})e^{i\mathbf{G}\mathbf{r}}\, d^2x = \Phi_0^2/2\pi|\mathbf{G}|$. The minimization is straightforward resulting in

$$n_{\mathbf{G}} = -\frac{\tilde{\epsilon}_v s_{\mathbf{G}}}{V_{\mathbf{G}}} = -\frac{2\pi\tilde{\epsilon}_v|\mathbf{G}|s_{\mathbf{G}}}{\Phi_0^2}, \tag{11}$$

$$U_v = -\frac{\pi\tilde{\epsilon}_v^2}{\Phi_0^2} \sum_{\mathbf{G}} |\mathbf{G}|\, |s_{\mathbf{G}}|^2. \tag{12}$$

Note that the solution becomes physically meaningless at positive $\tilde{\epsilon}_v$.

We apply these general results to analyze first the stripe domain structure. In this case the density of vortices $n(x)$ depends only on one coordinate x perpendicular to the domain walls. The vectors $\mathbf{G}$ are directed along the x axis. The allowed value of wave numbers are $G = \pi(2r + 1)/L$, where L is the domain width. The Fourier transform of the step function is $s_G = \frac{2i}{\pi(2r+1)}$. The inverse Fourier transform of Eq. (11) for the stripe domain case is

$$n(x) = -\frac{4\pi\tilde{\epsilon}_v}{\Phi_0^2 L}\frac{1}{\sin\frac{\pi x}{L}}. \qquad (13)$$

Note the strong singularity of the density near the domain walls. Our approximation is invalid at distances of the order of λ, and the singularities must be smeared out in the band of the width λ around the domain wall. Conversely, the approximation of the zero-width domain wall is invalid at least in the range of domain wall width l. Fortunately, we do not need more detailed information on the distribution of vortices in the vicinity of the domain walls. Indeed, by substituting the Fourier transform of the step function into Eq. (12), we find logarithmically divergent series. It must be cut off at $\pm r_{\max}$ with $r_{\max} \sim L/\lambda$. The summation can be performed using the Euler asymptotic formula [11] with the following result:

$$U_v^{\mathrm{str}} = -\frac{9\tilde{m}^2\mathcal{A}}{L}\left(\ln\frac{L}{\lambda} + C + 2\ln 2\right), \qquad (14)$$

where $\tilde{m} = m - 2\epsilon_v^0/3\Phi_0$. Now the problem of the proper cutoff for any lattice has to be analyzed. As we have seen already, the energy diverges logarithmically due to strong singularity of the vortex density near each domain wall [see Eq. (13)]. Thus, the logarithmic term is proportional to the total domain wall length. We need the next approximation, i.e., a term proportional to the length of domain wall without logarithmic factor. Such a term includes a nonlocal contribution from large distances between λ and L and a local contribution from the vicinity of the domain walls. The nonlocal contribution is accurately accounted for by the summation over the integers, whereas the local contribution requires a cutoff at large r, which is not well defined. However, due to its local character it must be the same for all domain walls. Therefore, it is possible to put the maximal wave vector in the direction normal to the domain wall to be, for example, $2\pi/\lambda$. Such a procedure results in the renormalization of the domain wall linear tension, the same for any domain lattice. This remark allows one to calculate the energy U_v for the square and triangular lattices.

In the case of the square checkerboard lattice, the allowed wave vectors are $\mathbf{G} = \frac{\pi}{L}[(2r+1)\hat{x} + (2s+1)\hat{y}]$. The Fourier transform of the step function is $s_{\mathbf{G}} = \frac{4}{\pi^2(2r+1)(2s+1)}$. The maximal values of r and s are identical and equal to L/λ, where L is the side of a square domain. The summation, similar to the case of stripe structure, although somewhat more complicated, leads to the following expression:

$$U_v^{\mathrm{sq}} = -\frac{18\tilde{m}^2\mathcal{A}}{L}\left(\ln\frac{L}{\lambda} + C + 2\ln 2 - \gamma\right), \qquad (15)$$

where the numerical constant γ is defined below:

$$\gamma = (2-\sqrt{2})\frac{7}{\pi^2}\zeta(3) + \frac{16}{\pi^2}\sum_{r=0}^{\infty}\sum_{s=r+1}^{\infty} S(r,s). \qquad (16)$$

Here $\zeta(x)$ is the Riemann zeta function; $\zeta(3) \approx 1.2020$ and $S(r,s) = [2(r+s+1) - \sqrt{(2r+1)^2 + (2s+1)^2}]/(2r+1)^2(2s+1)^2$. The direct numerical calculation gives $\gamma \approx 0.9 > \ln 2$.

The reciprocal lattice vectors for the regular triangular domain lattice are $\mathbf{G} = \frac{2\pi}{L}\{r[\hat{x} - (1/\sqrt{3})\hat{y}] + s(2/\sqrt{3})\hat{y}\}$. The analysis is remarkably simplified in the "triangular coordinate frame": $u = x - y/\sqrt{3}$; $v = 2y/\sqrt{3}$. The step function inside one elementary cell is $s(u,v) = +1$ for $u + v < L$ and $s(u,v) = -1$ for $u + v > L$, where L is the side of the elementary triangle. The Fourier transform of the step function $s_{\mathbf{G}}$ is not zero at either $r \neq 0, s = 0$, or $r = 0, s \neq 0$, or $r = s \neq 0$. For all these cases $|s_{\mathbf{G}}|^2 = 1/(\pi^2 q^2)$, where q is either r or s, depending on which of these numbers differs from zero. The summation in Eq. (12) in this case is rather simple resulting in

$$U_v^{\mathrm{tri}} = -\frac{72\mathcal{A}\tilde{m}^2}{L\sqrt{3}}(\ln r_{\max} + C). \qquad (17)$$

However, the value $r_{\max}$ is different from the stripe and square cases since the coordinates are skewed. Therefore, in this case $r_{\max} = \frac{\sqrt{3}}{2}\frac{L}{\lambda}$.

Our next step is to show that the magnetization self-interaction can be included into the renormalized domain wall linear tension. In the isolated FM, the magnetization self-interaction energy is equal to $U_{mm} = -m^2 L_{dw}\ln\frac{L}{l}$, where l is the domain wall width [12]. The superconducting screening enhances the magnetic field near the domain walls and reduces it outside a narrow vicinity of domain walls. In the stripe geometry $B_z^{(m)} = \frac{da}{dx} = -4\pi m\lambda\frac{d^2 s}{dx^2}$ implying that the screened magnetic field is confined in an interval $\sim\lambda$ near domain walls. Thus, its contribution to the energy does not contain a large logarithm: $U_{mm} \sim -L_{dw}m^2$ and can be incorporated into the renormalized value of domain wall linear tension. Note that this contribution is negative. We assume that it is less than the initial positive linear tension ϵ_{dw}. We do not consider here an interesting but less likely possibility of the negative renormalized linear tension, which probably results in the domain wall branching.

Now we are in position to minimize the total energy U over the domain width L and compare the equilibrium energy. The equilibrium domain width and the equilibrium energy for the stripe structure are

$$L_{\mathrm{eq}}^{(\mathrm{str})} = \frac{\lambda}{4}\exp\left(\frac{\varepsilon_{dw}}{9\tilde{m}^2} - C + 1\right), \qquad (18)$$

$$U_{\mathrm{eq}}^{(\mathrm{str})} = -\frac{36\tilde{m}^2\mathcal{A}}{\lambda}\exp\left(-\frac{\varepsilon_{dw}}{9\tilde{m}^2} + C - 1\right). \qquad (19)$$

Calculating the same values for the square and the triangular lattice, we obtain $L_{\mathrm{eq}}^{(\mathrm{sq})} = L_{\mathrm{eq}}^{(\mathrm{str})}\exp(\gamma)$; $U_{\mathrm{eq}}^{(\mathrm{sq})} = 2U_{\mathrm{eq}}^{(\mathrm{str})}\exp(-\gamma)$; and $U_{\mathrm{eq}}^{(\mathrm{tri})} = (3/4)U_{\mathrm{eq}}^{(\mathrm{str})}$. Comparing these energies to the energy of the stripe structure, we conclude that the stripe structure wins.

The domains become infinitely wide at $T = T_s$ and at $T = T_v$. The expression in the exponent (19) at $T = T_s$ is 9 times less than the corresponding expression for the domains in an isolated magnetic film. Therefore, the domains in the bilayer can be energetically favorable even if the isolated magnetic film remains in a monodomain state. If the domains in the magnetic film exist above the SC transition, they shrink dramatically below the transition. Bulaevsky and Chudnovsky [7] found that the domain width in a *thick* magnetic layer on the top of a *bulk* superconductor is proportional to $d_m^{1/3}$ instead of $d_m^{1/2}$, a well-known result for the isolated magnetic layer. Here d_m is the thickness of the magnetic layer; thick film means that d_m is much greater than the domain wall width l. Our problem is fundamentally different on two counts: first, we consider a *thin* FM film $d_m \ll l$ atop of the *thin* SC film, and second, the main effect stems from the interaction of vortices with the magnetization rather than from the magnetic field screening as in [7]. This effect is much stronger leading to a totally different dependence. If $\epsilon_{dw} \leq 9\tilde{m}^2$, the continuous approximation becomes invalid. Instead a lattice of discrete vortices must be considered, a problem that we will not address in the present publication. It is possible that the long nucleation time can interfere with the observation of described textures. We expect, however, that the vortices that appear first will reduce the barriers for domain walls and, subsequently, expedite domain nucleation. The quantitative study of this dynamic process is now in progress.

In conclusion, we predict that in a finite temperature interval below SC transition the FSB is unstable with respect to the SC vortex formation. The slow decay ($\propto 1/r$) of the long-range interactions between appearing Pearl vortices makes the structure that consists of alternating domains with opposite magnetization and vorticity energetically favorable. The distribution of vortices inside each domain is highly inhomogeneous, with increasing density closer to the domain walls. As long as the domain width is larger than the effective penetration depth, the energy of the stripe domain structure is minimal. These new topological structures can be observed directly. A strong anisotropy of the transport can be viewed as an indirect evidence of the stripe texture: The bilayer may be superconductive for a current parallel to the domains and resistive in the perpendicular direction.

We are grateful to O. Volpert for critical reading of the manuscript. This work was supported by NSF under Grants No. DMR 0103068 and No. DMR 0072115 and by DOE under Grants No. DE-FG03-96ER45598 and No. W-31-109-ENG-38.

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